TESTS OF THE STANDARD MODEL AND ITS EXTENSIONS IN HIGH-ENERGY e^+e^- COLLISIONS

RAIMO VUOPIONPERÄ

Research Institute for High Energy Physics
University of Helsinki
Helsinki, Finland

SEFT

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Preface

This Thesis is based on research carried out at the Department of High Energy Physics of University of Helsinki, at the Research Institute for High Energy Physics (SEFT), at Conseil Europeen pour la Recherche Nucleaire (CERN), at the High Energy Physics Laboratory of the Department of Physics of the University of Helsinki, and at the Deutsches Elektronen-Synchrotron (DESY). The work has been supported by the Lapin rahasto of the Finnish Cultural Foundation and the Vilho, Yrjö and Kalle Väisälä Foundation. My stay at DESY was funded by the Province of Finnish Lapland, the City of Rovaniemi and the municipality of Rovaniemen maalaiskunta. I wish to express my gratitude to these organizations, institutes and foundations.

Warm and loving thanks are due to my parents Petter and Estrid and to my brother Aarne and to my sister Linnea for their support and warmth. I thank also all my friends for their friendship and understanding.

This Thesis is dedicated to freedom and to all open minded human beings in our beautiful world.

Helsinki, March 5, 1996

Raimo Vuopionperä

Tests of the Standard Model and its extensions in high-energy e^+e^- collisions

Raimo Helmeri Vuopionperä
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Abstract

This thesis concerns the testing of the SM and its extensions in e^+e^- collisions, the main emphasis being on neutrino physics.

The future e^+e^- colliders will provide an excellent environment for precision tests of the Standard Model (SM) of particle interactions, as well as for a search of possible new phenomena going beyond the SM. The LEP upgrade will make it possible to test the self-interactions of the gauge bosons and other not so well known features of the SM, and at the planned TeV-range linear colliders one would be able to explore various extended schemes.

Besides the recently discovered top quark and the so far undetected Higgs particle, the tau neutrino (ν_τ) belongs to the most poorly known constituents of the model. A new concept for detecting the tau neutrino induced reactions in matter is presented. It is based on a very asymmetric e^+e^- collider combined with a large coarsely instrumented detector. Applying the same beam parameters as planned for the 500 GeV linear colliders, it is found that the tau neutrino would be detectable, though marginally, and its signature would be very clean.

The single top quark production at the LEP200 is also studied. While not important at LEP200, this process will play an important role as a t-quark source in the future e^+e^- linear colliders.

The pair production of heavy neutrinos and the single heavy neutrino production at e^+e^- colliders are also studied, and methods for distinguishing heavy Dirac and Majorana neutrinos are presented. At the 500 GeV e^+e^- linear collider the neutrinos with mass $m_N \lesssim 150$ GeV would be clearly detectable. It is shown that Dirac and Majorana neutrinos can be distinguished by using the angular distributions of the production processes and the production threshold behaviour of the heavy neutrino pair production.

The experimental implications of the left-right symmetric $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ model are also investigated. The single charged triplet Higgses $\Delta^\pm$ predicted by the left-right symmetric model are found to be observable at the upgraded e^+e^- collider with $\sqrt{s} = 1 - 2$ TeV and $\mathcal{L}_{\text{year}} \approx \mathcal{O}(100)$ fb^{-1}.

Contents

List of Papers

This Thesis consists of an introductory review part, followed by four research publications:

I: J. Maalampi, K. Mursula and R. Vuopionperä,
Heavy neutrinos in e^+e^- collisions,
Nuclear Physics B **372** (1992) 23–43.

II: R. Vuopionperä,
Heavy Dirac and Majorana neutrino production in e^+e^- collisions,
Zeitschrift für Physik C **65** (1995) 311–325.

III: R. Keränen, J. Pennanen and R. Vuopionperä,
How to observe $\nu_\tau N$ interactions at an extremely asymmetric e^+e^- collider,
Physical Review D **46** (1992) 4852–4855.

IV: M. Raidal and R. Vuopionperä,
Single top quark production at LEP200,
Physics Letters B **318** (1993) 237–240.

1 Introduction

The serious and systematic study of the structure of matter can be said to have started at the end of the 19th century, and it has continued ever since with an increasing pace. Experimental methods for investigating the structure of matter have developed from tabletop atomic physics measurements, such as the experiments of Rutherford, to the international multi-million dollar particle physics experiments in huge present day accelerators. At the same time theories describing the basic laws of Nature have reached very high level in mathematical rigor.

The development of physical theories and the construction of experimental apparatuses have sometimes been rather loosely connected with each other. However, in high-energy physics, where experiments nowadays are very large and expensive, it has been realized that one cannot construct accelerators and detectors without having a good understanding of the nature of the phenomena one wants to investigate. This is the attitude followed also in this Thesis. We investigate the phenomenology of electroweak interactions at future high energy colliders and also propose some new experimental methods to study the constituents of various particle physics models.

Ever since conjectured by W. Pauli in 1930 [1], neutrinos have played an important role in our understanding of the basic laws of particle physics. Neutrino was originally proposed to maintain the energy conservation law in nuclear beta-decay [2]. One can see, in a sense, a direct route from the neutrino hypothesis to the present gauge theories of electroweak interactions. The first step along this route was taken by Fermi who formulated a field theoretical description for neutrino interactions [3]. In the Fermi theory neutrinos were assumed massless, as was also discussed by Perrin [4]. A quarter of century later neutrino interactions were found to be of the $V - A$ form [5], and the predicted parity violation was discovered in beta decay [6].

The existence of the electron neutrino was confirmed in 1956 by Reines and Cowan

[7], who used an antineutrino beam from a nuclear pile to detect the reaction $p^+ + \bar{\nu} \to$ $n + e^+$ followed by the reaction $e^+ + e^- \to \gamma + \gamma$ and a γ ray emitted from neutron capture on ^{35}Cd in a large mass scintillator. The fact that neutrinos emitted in beta decay are left-handed, as the $V - A$ theory predicts, was confirmed by an angular correction measurement with ^{35}A [8], and also in an experiment which measured the circular polarisation of the γ rays emitted by the exited state of ^{152}Sm in the electron capture of ^{152}Eu [9]. The existence of s second neutrino, the muon neutrino, was confirmed by Dandy et al. in 1962 [10].

Long before the so-called Standard Model (SM) of electroweak interactions [11] was formulated in its present form, it was understood that the Fermi theory is just a low-energy effective theory [12]. The experimental verification of the SM, whose renormalizability was proven by 't Hooft [13], was obtained via the discovery of neutrino induced neutral current reactions [14] and finally by the discovery of the $W^{\pm}$ and Z^0 bosons [15]. Despite the fact that there are some unsatisfactory features in SM, by now the model is experimentally very precisely verified [16, 17, 18]. After the recent discovery of the top quark at TEVATRON [18], the only missing constituents of the SM are the Higgs particle and the tau neutrino (ν_τ). Reactions induced by the tau neutrino have not so far been directly observed. On the other hand, experiments at the $e^+ e^-$ collider LEP at CERN have shown the number of light neutrinos to be three [16]. The decay spectrum in some decay channels of the tau-lepton also indirectly indicates the existence of the tau neutrino [19]. The direct discovery of the tau neutrino is a topical issue [20], also addressed in this Thesis.

Neutrinos are the only particles in the SM which are sensitive to only one type of interaction, the weak force. Therefore neutrinos may play an important role in revealing possible new physics phenomena, which for other particles might be shadowed by the effects of electromagnetic and strong interactions. One important question, where new physics may manifest itself, is neutrino mass. The ordinary neutrinos are known to be very light ($m_{\nu_e} \lesssim \mathcal{O}(10)$ eV, $m_{\nu_\mu} \lesssim 270$ keV, $m_{\nu_\tau} \lesssim 31$ MeV [21]) as compared with other fermions in the same fermion generation. Finite neutrino masses would indicate physics beyond the SM, since in the SM neutrinos are strictly massless.

The precision experiments have confirmed that the gauge interactions of light

neutrinos are quite precisely described by the SM (see e.g. [22]). Apart from the existence of neutrino mass, these experiments leave also many other questions unanswered. These include the question of lepton number violation [23] and the related question of the nature of neutrino, i.e. is it a Dirac or a Majorana particle [24], the neutrino mixing, as well as the possible CP violation in the lepton sector. If neutrinos have a non-zero mass one must ask whether they are stable particles or not. For very heavy neutrinos, which have not been discovered so far, the nature of gauge couplings is also an issue one should consider. The light neutrinos studied in many experiments are known to have left-handed $V - A$ interactions, but the gauge couplings of the heavy neutrinos can a priori have a more general vector - axial-vector structure as a reflection of the structure of a possible underlying gauge theory.

From the experimental point of view, the search for small finite neutrino masses, together with the generation mixing among the light neutrinos, has so far got the most of the attention [25]. The future accelerators, such as the planned e^+e^- linear colliders [26, 27], will offer a good environment for direct searches of possible heavy neutrinos and for the study of their properties. This is one of the main topics of this Thesis. The search of heavy neutrinos in proton colliders and in electron-proton colliders has been considered, e.g., in [28] and [29].

There are plenty of models of electroweak interactions which predict heavy neutrinos. The simplest of such models is obtained by adding right-handed heavy neutrinos to the SM particle spectrum as $SU(2)$ singlets. If the right-handed singlets are not present, but neutrinos have a mass, then they are necessarily Majorana particles. This is the situation in the SM as well as in the grand unified theories based on $SU(5)$ symmetry [30]. In the left-right symmetric model (LR-model) [31], whose gauge symmetry is $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$, one has heavy Majorana neutrinos and in some versions of the model also heavy Dirac neutrinos [32]. The existence of these heavy states is related to the so-called see-saw mechanism [33], in terms of which one can explain the tiny mass of the left-handed neutrinos in a very attractive way. The LR model is naturally embedded in $SO(10)$-based grand unified theories (GUT) [34], which have the nice feature that all the fermions of a given family, including the right-handed neutrinos, are assigned into an irreducible anomaly free representation. Among many

other models that predict heavy neutrinos one should mention the superstring inspired E_6 models [35].

This Thesis is organized as follows. First we give a brief summary of the appended original research Papers. In Section 2 we describe the central elements of the Standard Model, in particular its particle spectrum from the point of view of neutrino physics. Some aspects of the SM phenomenology at the e^+e^- colliders are also briefly discussed in this Section. Section 3 deals with extended gauge models. The physics of heavy neutrinos in the left-right symmetric electroweak model is studied in more detail. In Section 4 we present our conclusions.

Summary of the Original Papers

Paper I: Heavy neutrinos in e^+e^- collisions. In this Paper the production and the subsequent decay of heavy neutrinos in e^+e^- collision are systematically investigated. The purpose is to study the possibility of distinguishing heavy Dirac and Majorana neutrinos (N) from each other in electron-positron collisions. In the previous studies [36] the main attention has been paid on the nature of the *light* neutrinos. A new experimentally easy method for separating the Dirac and Majorana cases is presented. It is based on equal-sign lepton pair correlations in the decay of $N\bar{N}$ system. This method would work well even with low statistics. General formulas for the pair and single heavy neutrino production cross sections and the decay widths of heavy neutrinos are derived. Also other experimental methods for distinguishing Dirac and Majorana cases are discussed on the basis of these formulas. The experimental signals of the heavy neutrino production are shown to be clean and almost background free, especially in the νN production channel where monojets and otherwise strongly unbalanced events will appear.

Paper II: Heavy Dirac and Majorana neutrino production in e^+e^- collisions. This Paper broadens the analysis of the Paper I by considering the production of heavy neutrinos within a more general theoretical framework. The effects of non-standard Higgs bosons to the heavy neutrino production in electron-positron collisions are investigated systematically. The analytical differential cross section formulas are derived using the most general Lagrangian with complex lepton and boson couplings. For the total cross sections the numerically integrated results are shown. All the interference terms are carefully studied and they are found to be fairly large in a wide range of the production spectrum. Using a simple version of the left-right symmetric model, specified in the Paper, the heavy neutrino signal is found to be clearly visible at a 500 GeV linear e^+e^- collider with a luminosity around $\mathcal{L}_{\text{year}} = 10 \text{ fb}^{-1}$. Even the detection of the triplet higgs $\Delta^\pm$ with a mass around 1 TeV is found marginally possible

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if a second phase of the e^+e^- collider with a higher collision energy ($\sqrt{s} = 1.0 - 2.0$ TeV) and a higher luminosity ($\mathcal{L}_{\text{year}} \approx 100$ fb^{-1}) is constructed after the preliminary 0.5 TeV phase.

Paper III: How to observe $\nu_\tau N$ interactions at an extremely asymmetric e^+e^- collider. The weakly interacting partner of the tau lepton, the tau neutrino, has not been directly observed, although its existence is indirectly verified through τ decays [19]. Also results of the LEP and SLC have confirmed that in addition to the electron neutrino and the muon neutrino a third light neutrino species exists. In this Paper a new and novel idea for an accelerator and detector concept for a direct tau neutrino discovery is proposed. It is shown that by using a very asymmetric ring-linac type [37] electron and positron collider one could produce a well-focused and monoenergetic tau neutrino beam. A robust layout for a whole accelerator and detector setup is presented and the most critical technical aspects are studied. Using Monte Carlo simulation a method to separate the signal from background is presented. The luminosity required for the tau neutrino detection is found to be in the range $\mathcal{L} \approx 10^{34}$ cm^{-2}s^{-1}, i.e. comparable with the luminosity of the proposed electron-positron colliders [26, 27].

Paper IV: Single top quark production at LEP200. In this Paper the prospects for detecting the top quark at the LEP200 collider are investigated. It is argued that it would be marginally possible to observe the single top production at LEP200. The cross section of the reaction $e^+e^- \rightarrow t\bar{b}e^-\bar{\nu}_e$ ($\bar{t}be^+\nu_e$), is estimated by integrating the analytical eight dimensional differential cross sections by using Monte Carlo integration method. Later more detailed numerical and analytical studies by other authors [38] have shown, however, that our results overestimate the production rate and our conclusions may therefore be too optimistic as far as LEP200 is concerned. For higher energy e^+e^- collisions the process considered will be an important source of the top quarks.

2 The Standard Model

The Weinberg-Salam model [11] is the most successful theory for the electroweak interactions of quarks and leptons. It is a renormalizable field theory based on the $SU(2)_L \otimes U(1)_Y$ gauge symmetry spontaneously broken to the residual $U(1)_{em}$ symmetry of electromagnetic interactions. To implement the spontaneous breaking one has to include scalar particles, Higgs bosons, to the particle spectrum of the model. The spontaneous symmetry breaking generates masses of the weak bosons $W^{\pm}$ and Z^0 through the so-called Higgs mechanism [39], and at the same time masses of quarks and leptons via the Yukawa couplings of scalars and fermions. The strong interactions of quarks described by the quantum chromodynamics (QCD) are included to the model by adding an $SU(3)_C$ gauge symmetry to the Weinberg-Salam symmetry, which then together form the Standard Model (SM).

The particle spectrum of the SM consists of scalar Higgs particles, vector bosons and spin-1/2 fermions. The fermions, i.e. leptons and quarks, are grouped into three fermion generations. Under the $SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$ gauge symmetry each fermion generation ($k = 1, 2, 3$) has the following field contents (the color indices of the quark fields are suppressed):

$$
\begin{aligned}
L_k &= \begin{pmatrix} \nu_{\ell_k} \\ \ell_k \end{pmatrix}_L \sim (2, -1, 1), \quad \ell_{kR} \sim (1, -2, 1), \\
Q_{kL} &= \begin{pmatrix} u_k \\ d_k \end{pmatrix}_L \sim (2, \tfrac{1}{3}, 3), \quad u_{kR} \sim (1, \tfrac{4}{3}, 3), \quad d_{kR} \sim (1, -\tfrac{2}{3}, 3).
\end{aligned}
\tag{1}
$$

The right-handed neutrinos, not present in the minimal version of the SM, would be totally inert with respect to gauge interactions, i.e. they would transform as $\nu_{kR} \sim (1, 0, 1)$.

The scalar sector of the SM consists of a Higgs doublet

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (2, 1, 1), \tag{2}$$

where the neutral component acquires vacuum expectation value, thereby breaking the electroweak symmetry. This is the minimal case, but in general the Higgs sector may be more complicated. For example, in the minimal supersymmetric version of the SM (MSSM) [40] there should exist two scalar doublets, at least. One can also add different scalar representations (e.g. $SU(2)_L$-triplets) to the scalar particle spectrum, as will be described later on.

The Lagrangian of the SM is of the form

$$\mathcal{L} = \mathcal{L}_{kin}^{matter} + \mathcal{L}_{kin}^{gauge} + \mathcal{L}_Y - V(\phi). \tag{3}$$

The kinetic terms are given by

$$
\begin{aligned}
\mathcal{L}_{kin}^{matter} &= \left| \left(\partial_\mu - i\frac{g}{2}\overline{\tau} \cdot \overline{W}_\mu - i\frac{g'}{2}B_\mu \right) \phi \right|^2 \\
&\quad + i\,\bar{Q}_{kL}\gamma^\mu \left(\partial_\mu - i\frac{g}{2}\overline{\tau} \cdot \overline{W}_\mu - i\frac{g'}{6}B_\mu \right) Q_{kL} \\
&\quad + i\,\bar{u}_{kR}\gamma^\mu \left(\partial_\mu - i\frac{2g'}{3}B_\mu \right) u_{kR} + i\,\bar{d}_{kR}\gamma^\mu \left(\partial_\mu + i\frac{g'}{3}B_\mu \right) d_{kR} \\
&\quad + i\,\bar{L}_k\gamma^\mu \left(\partial_\mu - i\frac{g}{2}\overline{\tau} \cdot \overline{W}_\mu + i\frac{g'}{2}B_\mu \right) L_k \\
&\quad + i\,\bar{\ell}_{kR}\gamma^\mu \left(\partial_\mu + i\,g'B_\mu \right) \ell_{kR} + i\,\bar{\nu}_{kR}\gamma^\mu \partial_\mu \nu_{kR}
\end{aligned} \tag{4}
$$

and

$$\mathcal{L}_{Kin}^{Gauge} = -\frac{1}{4}\overline{W}_{\mu\nu} \cdot \overline{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{5}$$

where

$$\overline{W}_{\mu\nu} = \partial_\mu \overline{W}_\nu - \partial_\nu \overline{W}_\mu + g\,\overline{W}_\mu \times \overline{W}_\nu, \tag{6}$$

and $\overline{W}_\mu$ stands for $(W_{1\mu}, W_{2\mu}, W_{3\mu})$. We have included in the Lagrangian (4), and in what follows, also the right-handed neutrinos to serve our later considerations.

The Yukawa terms describing interactions of the quarks and leptons with the Higgs scalars are given by the Lagrangian

$$-\mathcal{L}_Y = (h_d)_{kj} \, \bar{Q}_{kL}\phi d_{jR} + (h_u)_{kj} \, \bar{Q}_{kL}\tilde{\phi}u_{jR} + (h_\ell)_{kj} \, \bar{L}_k\phi\ell_{jR} + (h_\nu)_{kj} \, \bar{L}_k\tilde{\phi}\nu_{jR} + h.c. \,, \quad (7)$$

where h_d, h_u, h_ℓ and h_ν are dimensionless Yukawa coupling matrices and the charge conjugate of the scalar field is defined as $\tilde{\phi} = i\tau_2\phi^* \sim (2, -1, 1)$. The last term of (7) gives rise to neutrino masses via the spontaneous symmetry breaking. In the minimal SM it does not exist, and the neutrinos are massless.

The Higgs potential is chosen to be

$$V(\phi) = -\mu^2(\phi^\dagger\phi) + \frac{\lambda}{4}(\phi^\dagger\phi)^2 \,, \quad \mu^2 > 0. \quad (8)$$

The minimum of this potential corresponds to the vacuum expectation value

$$\langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad (9)$$

of the scalar doublet, where $v = 2\mu/\sqrt{\lambda}$. The vacuum breaks the electroweak symmetry $SU(2)_L \otimes U(1)_Y$, except the $U(1)_{em}$ subsymmetry associated with the electromagnetism. The gauge bosons, $W^\pm$ and Z^0, corresponding to the broken generators acquire mass [11], while the gauge boson, photon, corresponding to the unbroken generator of $U(1)_{em}$ remains massless. In other words, the SM symmetry $SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$ is spontaneously broken down to the $U(1)_{em} \otimes SU(3)_C$ residual gauge symmetry. The masses of the weak bosons $W^\pm$ and Z^0 are

$$M_W = \frac{gv}{2} \,, \quad M_Z = \frac{v\sqrt{g^2 + g'^2}}{2} = \frac{M_W}{\cos\theta_W} \,, \quad (10)$$

where θ_W is the so-called weak mixing angle.

In terms of the physical gauge fields the interaction Lagrangian is given by

$$\mathcal{L}_{NC} = -eJ_{em}^\mu A_\mu - GJ_{NC}^\mu Z_\mu = -e\, J_{em}^\mu A_\mu - G\left(J_3^\mu - \sin^2\theta_W J_{em}^\mu\right) Z_\mu \,, \quad (11)$$

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}} \, J_{CC}^\mu W_\mu^+ + h.c. \,, \quad (12)$$

where

$$G^2 = g^2 + g'^2 \,, \tag{13}$$

$$J^\mu_{NC} = J^\mu_3 - \sin^2\theta_W J^\mu_{em} \,, \tag{14}$$

$$J^\mu_{CC} = 2\left[\bar\nu_{kL}\gamma^\mu \ell_{kL} + \bar u_{kL}\gamma^\mu d_{kL}\right] \,, \tag{15}$$

$$J^\mu_{em} = -\bar\ell\gamma^\mu\ell + \frac{2}{3}\bar u\gamma^\mu u - \frac{1}{3}\bar d\gamma^\mu d \,, \tag{16}$$

$$J^\mu_3 = -\frac{1}{2}\bar\ell_L\gamma^\mu\ell_L + \frac{1}{2}\bar\nu_L\gamma^\mu\nu_L - \frac{1}{2}\bar u_L\gamma^\mu u_L + \frac{1}{2}\bar d_L\gamma^\mu d_L \,. \tag{17}$$

The weak mixing angle θ_W has the experimentally measured value $\sin^2\theta_W^{LEP} = 0.2319 \pm 0.0005 \pm 0.0002$ [41].

The quark and lepton fields appearing in the above interaction Lagrangian, so-called weak eigenstates, do not in general correspond to the physical propagating particle states with a definite mass. The mass eigenstates are obtained by diagonalizing the Yukawa Lagrangian (7) by suitable unitary transformations. For example, the physical up-type quarks u', c', t' are given in terms of the interaction eigenstates as follows:

$$
\begin{aligned}
\begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}_L &= U_u \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L \,, \\[2em]
\begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}_R &= V_u \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R \,,
\end{aligned}
\tag{18}
$$

where $U_u^\dagger U_u = V_u^\dagger V_u = 1$.

Only the form of the charged current part of the gauge Lagrangian (12) is changed when one moves from the interaction basis of the fermion fields into the mass basis. It reads now

$$J^\mu_{CC} = 2\left[\bar\nu'_{jL}(U^{CKM}_\ell)_{jk}\gamma^\mu\ell'_{kL} + \bar u'_{jL}(U^{CKM}_q)_{jk}\gamma^\mu d'_{kL}\right] \,, \tag{19}$$

where the complex unitary Cabibbo-Kobayashi-Maskawa mixing matrices are $U^{CKM}_\ell = U^\dagger_\nu U_l$ and $U^{CKM}_q = U^\dagger_u U_d$. The U^{CKM}_q matrix, except for the U_{tx} elements, is experimentally quite well known [42]. Even the U_{tx} elements can be fairly well limited on

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the grounds of the unitarity of U_q^{CKM}. In contrast, the elements of the lepton mixing matrix U_ℓ^{CKM} are almost unknown. The results of neurinoless beta decay experiments give only weak constraints on U_ℓ^{CKM} [43].

Obviously the mixing formalism is valid for any number n_f of fermion generations, and the number of the quark and the lepton generations may also differ in a general case. The $n_f \times n_f$ unitary matrices can be parameterized with n_f^2 real parameters, i.e. with $\frac{1}{2}n_f(n_f - 1)$ rotational angles and $\frac{1}{2}n_f(n_f + 1)$ phases. In the case of fermion mixing, however, $2n_f - 1$ phases can be absorbed by redefining the physical fermion fields and therefore the CKM matrices can be parameterized in terms of $\frac{1}{2}n_f(n_f - 1)$ rotational angles and $\frac{1}{2}(n_f - 1)(n_f - 2)$ phases [44]. In the case of three generations this means three rotation angles and one phase.

The non-diagonal nature of the charged currents permits flavor changing reactions, such as $K^+ \to \mu^+\nu_\mu$, where one has the transition $\bar{s} \to \bar{d}$ [45], and the CP violating reactions, such as $K_L^0 \to \pi^+\pi^-$ [46], arise because of the complex phase in the mixing matrix. Flavor changing neutral current and CP violating reactions have been observed in the quark sector [47], but never in the lepton sector, where the very small neutrino mass forces the corresponding decay widths and cross sections to be negligible. If neutrinos do not have a mass there is neither flavor changing neutral currents nor CP violating reactions in the lepton sector because the mass eigenstates of the leptons are in this case the same as the current eigenstates.

2.1 Neutrino Masses in $SU(2)_L \otimes U(1)_Y$ Model

If the right-handed neutrino fields exist, the mass generation mechanism described above for quarks and charged leptons would also give rise to massive neutrinos in the SM. Such massive neutrinos will be Dirac particles. In this case the physics of the lepton sector would be very similar to that of quarks. In particular, the lepton generation mixing is analogous to the quark generation mixing, and the mixing matrix

could be presented in the general form

$$U_{q(\ell)}^{CKM} = \begin{pmatrix} c_{\theta_2}c_{\theta_1} & c_{\theta_2}s_{\theta_1} & s_{\theta_2} \\ -c_{\theta_3}s_{\theta_1}e^{i\delta}-s_{\theta_3}s_{\theta_2}c_{\theta_1} & c_{\theta_3}c_{\theta_1}e^{i\delta}-s_{\theta_3}s_{\theta_2}s_{\theta_1} & s_{\theta_3}c_{\theta_2} \\ s_{\theta_3}s_{\theta_1}e^{i\delta}-c_{\theta_3}s_{\theta_2}c_{\theta_1} & -s_{\theta_3}c_{\theta_1}e^{i\delta}-c_{\theta_3}s_{\theta_2}s_{\theta_1} & c_{\theta_3}c_{\theta_2} \end{pmatrix}. \tag{20}$$

As a result of the mixing the flavor changing charged lepton currents will hence be allowed.

The mass mixing would also make possible the neutrino oscillations [48, 49], i.e.

$$\nu_\ell \leftrightarrow \nu_{\ell'} \quad \text{and} \quad \nu_\ell^c \leftrightarrow \nu_{\ell'}^c \quad (\ell \neq \ell') \,. \tag{21}$$

On the other hand, neutrino-antineutrino oscillations $\nu_\ell \leftrightarrow \nu_{\ell'}^c$ are forbidden. This is because the total lepton number $L = \sum_{i=1}^n L_i$ is conserved by the Dirac mass terms.

If there are no right-handed neutrinos in the theory, the above mass generation mechanism does not work. The only allowed mass term in this case is the so-called Majorana mass term

$$\mathcal{L}_\nu^M = \left(m_\nu^M\right)_{ij} \nu_{iL}^T C \nu_{jL} + \text{h.c.} \equiv \left(m_\nu^M\right)_{ij} \bar{\nu}_{iR}^c \nu_{jL} + \text{h.c.}\,, \tag{22}$$

where C is the charge conjugation matrix and $\bar{\nu}_R^c = \nu_L^T C$. As such this would violate the gauge invariance and is therefore not present in the original Lagrangian, but it may enter as a result of a spontaneous symmetry breaking involving an $SU(2)_L$ triplet scalar multiplet $\Delta \sim (3, 2, 1)$. The corresponding Yukawa term is

$$\begin{aligned} \mathcal{L}_Y'' &= (h_\ell'')_{ij} \left(\nu_{iL}^T \; \ell_{iL}^T\right) C\,(i\sigma_2)\,\vec{\sigma} \begin{pmatrix} \nu_{jL} \\ \ell_{jL} \end{pmatrix} \cdot \vec{\Delta} + \text{h.c.} \tag{23} \\[2mm] &= (h_\ell'')_{ij} \left(-\bar{\ell}_{iR}^c \; \bar{\nu}_{iR}^c\right) \begin{pmatrix} \Delta_3 & \Delta_1 - i\Delta_2 \\ \Delta_1 + i\Delta_2 & -\Delta_3 \end{pmatrix} \begin{pmatrix} \nu_{jL} \\ \ell_{jL} \end{pmatrix} + \text{h.c.} \tag{24} \\[2mm] &= (h_\ell'')_{ij} \left[-\frac{1}{\sqrt{2}}\left(\bar{\ell}_{iR}^c \nu_{jL} + \bar{\nu}_{iR}^c \ell_{jL}\right)\Delta^+ - \bar{\ell}_{iR}^c \ell_{jL}\Delta^{++} + \bar{\nu}_{iR}^c \nu_{jL}\Delta^0\right] + \text{h.c.} \tag{25} \\[2mm] &= -\left(h_+'\right)_{ij}\bar{\ell}_{iR}^c \nu_{jL}\Delta^+ - \left(h_+''\right)_{ij}\bar{\nu}_{iR}^c \ell_{jL}\Delta^+ \\ &\quad - \left(h_{++}'\right)_{ij}\bar{\ell}_{iR}^c \ell_{jL}\Delta^{++} + (h_0')_{ij}\bar{\nu}_{iR}^c \nu_{jL}\Delta^0 + \text{h.c.}\,, \tag{26} \end{aligned}$$

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where $\Delta^0 = \Delta_1 + i\Delta_2$, $\Delta^+ = \sqrt{2}\,\Delta_3$ and $\Delta^{++} = \Delta_1 - i\Delta_2$. The last equation (26) is the most general form of this kind of Yukawa coupling terms. The Majorana mass term (22) arises when the vacuum expectation value $\langle\Delta^0\rangle = v_L \neq 0$, and the Majorana neutrino mass matrix of the Standard Model appearing in (22) is thus

$$\left(m_\nu^M\right)_{ij} = (h'_0)_{ij}\left\langle\Delta^0\right\rangle . \tag{27}$$

It should be emphasised that the spontaneous breaking of the $SU(2)_L \otimes U(1)_Y$ symmetry does not require the existence of a triplet Higgs Δ. Actually, the experimentally measured value of the parameter $\rho_0 = (M_W/\cos\theta_W M_Z)^2 = 1.0004 \pm 0.0022 \pm 0.002$ [50] indicates that the vacuum expectation value of the left-handed triplet Higgs field should be much smaller than that of the doublet scalar field ϕ, i.e. $\langle\Delta_L\rangle \lesssim \mathcal{O}(10)$ GeV $\ll \langle\phi\rangle \equiv v/\sqrt{2} \simeq 175$ GeV. Hence, the existence of the triplet Δ_L is not strongly motivated, and one can say that neutrinos are most naturally massless in the $SU(2)_L \otimes U(1)_Y$ model.

The generation mixing in the case of Majorana neutrinos differs from the quark generation mixing in many respects. Due to Fermi statistics the mass matrix must be symmetric $M \equiv \left(m_\nu^M\right) = \left(m_\nu^M\right)^T$. One can diagonalize such a symmetric and complex mass matrix by the following transformation:

$$m = \mathrm{diag}(m_1, \ldots, m_n) = U^T M U , \tag{28}$$

where $m_i \geq 0$ and U is unitary matrix. In contrast to the Dirac mass case, only one unitary matrix is needed to diagonalize a Majorana mass matrix. This is of course a consequence of the fact that the Majorana mass term involve just the left-handed fields. The mass Lagrangian can be rewritten in the form

$$\mathcal{L}_\nu^M = -\frac{1}{2}\bar{\chi}m\chi = -\frac{1}{2}\sum_{i=1}^n m_i\bar{\chi}_i\chi_i \tag{29}$$

$$= -\frac{1}{2}\left(\bar{n}_R^c m n_L - \bar{n}_L m n_R^c\right) = -\frac{1}{2}\left(n_L^T C m n_L - n_R^{c\,T} C n_R^c\right) , \tag{30}$$

where $n_L = U^\dagger\nu_L$, $n_R^c = C\bar{n}_L^T = U^T\nu_R^c$ and

$$\chi = n_L + n_R^c = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \vdots \\ \chi_n \end{pmatrix} . \tag{31}$$

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The chiral composition of the mass eigenstates χ_i is the following:

$$\chi_i = \chi_{iL} + \chi_{iR} = n_{iL} + n_{iR}^c = \sum_{j=1}^{n} \left[\left(U^\dagger \right)_{ij} \nu_{jL} + \left(U^T \right)_{ij} \nu_{jR}^c \right]. \tag{32}$$

The states obey $\chi_i = \chi_i^c = C\bar{\chi}_i^T$, i.e. they are Majorana particles.

The current states are given in terms of mass eigenstates via the formula

$$\nu_{jL} = \sum_{i=1}^{n} \left(U \right)_{ji} \chi_{iL}. \tag{33}$$

The functional form of the charged current Lagrangian for the mass eigenstates χ_i is exactly the same as in the Dirac case (eq. (19)), except that the ν_i' fields are replaced by the χ_i fields. The neutral current term (see eqs. (14), (16) and (17)) remains flavor diagonal, but the physical ν_i' fields are replaced by χ_i fields.

In the Majorana case the total lepton number is an invariant in the charged and neutral current gauge interactions but the Majorana mass term breaks the lepton number by two units. So, in the case of the Majorana neutrinos both the neutrino-neutrino oscillations and the neutrino-antineutrino oscillations are possible.

2.2 Standard Model Phenomenology at $e^- e^+$ Collisions

Some of the features of the SM have not been directly confirmed by measurements yet. Apart from the Higgs particle, the only undiscovered constituent of the SM particle spectrum is the neutral partner of the τ lepton, the tau neutrino ν_τ. The heaviest quark, the t-quark, another member of third fermion generation, has recently been discovered at Tevatron [18]. The mass and the other characteristics of the top quark are still quite poorly known.

The experimental tests of the SM predictions have by now reached the level of a few per mils [51]. While most of the recent particle discoveries have been made by using hadron colliders, e.g. the discovery of the W and Z bosons at SPS and the top quark at Tevatron, the precision tests are mainly due to lepton collision experiments. Due to the overwhelming background problems at future hadron experiments, one expects the role of lepton machines to become more important in searching new physics in

the future. The discovery of the SM Higgs particle, not considered in this Thesis, will obviously be one of the main goals of the future e^+e^- colliders.

In Papers III and IV we study possibilities to investigate the tau neutrino and the top quark in experiments with lepton beams. In Paper III a new idea of using a very asymmetric e^+e^- collider to create a monoenergetic tau neutrino beam of high intensity is proposed. Asymmetric e^+e^- collisions are achieved by using electron and positron beams with unequal beam energies. Adjusting the beam energies properly one can produce $\nu_\tau\bar{\nu}_\tau$ and $\tau^+\tau^-$ pairs in a highly boosted CM frame. Thus, most of the tau neutrinos are boosted to very narrow cone in the laboratory frame. The energy and the production angle of neutrinos are correlated. For observing the tau neutrino via the reactions it induces in matter, we propose a long coarsely instrumented iron spectrometer.

The feasibility of the proposed experimental setup is studied using computer simulations which took into account the physical processes and the experimental constraints. The signal is found to be marginally observable if the high-energy beam parameters, e.g. the beam size and beam intensity at the interaction point, designed for the future e^+e^- colliders, are used. On the other hand, the signal is found to be almost background free due to the experimental setup which is designed in such a way that only neutrinos can enter the detector. If the luminosity of the accelerator and/or the volume of the detector are further increased, the expected signal will, of course, be enhanced. For detecting the tau neutrino also other methods have been considered in the literature, such as beam dumb experiments [52] and the proposed tau neutrino experiment at the LHC [20]. As compared with these, the arrangement we propose has the virtue of giving an almost monoenergetic neutrino beam and it also has a much better signal to background ratio.

The main focus of Paper IV is the top quark. The work was done prior to the discovery of the top quark at Tevatron [18]. We investigated the possibilities discovering and studying the top quark at the upgraded LEP200 via the single top production reaction $e^+e^- \to t\bar{b}e^-\bar{\nu}_e$ ($\bar{t}be^+\nu_e$). The total number of tree level amplitudes for this reaction is thirty, but most of them give a negligible contribution. We included in our calculations eleven graphs. Due to the very small Yukawa coupling between the Higgs

boson and the electron, the processes involving a neutral Higgs are negligible and are not taken into account. Due to the high mass of the Z^0 boson the phase space is rather limited and thus the production rates are strongly reduced in the Z^0 exchange reactions compared with the dominant photon processes (see ref. [53]).

The results of our calculations made us to conclude that it would be marginally possible to observe the top quark at the upgraded LEP200. Later and more detailed studies by other authors [38] have shown, however, that our production rate estimates are inaccurate and for that reason too optimistic. The origin of the inaccuracy of our analysis can be traced back to large cancellations between different individual processes which caused undetectable numerical errors in our Monte Carlo integration program.

Thanks to the efforts of many authors [38] the single top production process at e^+e^- collisions is now totally under control and well understood. While not important at LEP200, this process will play an important role as a t-quark source in the future high-energy e^+e^- colliders [54].

3   Extended Gauge Models

In spite of its theoretical appeal and excellent phenomenological success, the Standard Model has some unsatisfactory features. It suffers from the naturalness problem [55] and the hierarchy problem [56] and, furthermore, it does not give any explanation for the the maximal parity violation [57] or for the origin of CP violation [58]. There is also a great number of free parameters, 19 altogether, which one can fix only by experimental measurements. Also, the SM does not explain the family structure of the fermions.

Some of these shortcomings seem to point towards a further unification of electroweak and strong interactions in so-called grand unified theories (GUT's), some towards supersymmetry [59], compositeness [60] or so-called technicolor schemes [61]. The most studied GUT models are the $SU(5)$ and $SO(10)$ theories, and due to the LEP results [62] the supersymmetric versions of these models seem to be favored at the moment. The preonic composite models have been recently studied extensively in the literature [63]. The technicolor models and similar composite models [64] are somewhat disfavored by current experimental results (see e.g. [65]).

In this Thesis we will mainly concentrate on the gauge models of electroweak interactions based on a larger gauge group than the $SU(2)_L \otimes U(1)_Y$ of the SM. In general, the only requirements we impose on such an extended gauge model is that it should contain all the SM particles and that its gauge symmetry must eventually break down to the SM gauge group. In general, we are also interested in unified theories which, unlike the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetric SM, does unify all the forces within one gauge group. All this can be accomplished using higher symmetry groups which contain the SM gauge group as a subgroup.

The breaking of the grand gauge symmetry (G_U) to the SM symmetry proceeds

in general through several intermediate stages:

$$G_U \overset{M_U}{\to} G_{I_1} \overset{M_{I_1}}{\longrightarrow} \dots \overset{M_{I_{n-1}}}{\longrightarrow} G_{I_n} \overset{M_{I_n}}{\to} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \,, \qquad (34)$$

where the intermediate breakings occur at energy scales M_U and $M_{I_1}, \dots, M_{I_n}$.

In this Thesis particular attention is paid to such extended gauge models which at some stage of the breaking chain contain an $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$ gauge symmetry, the so-called left-right electroweak symmetry [31]. Otherwise the considered extended gauge models are only restricted by the experimental results, i.e. new interactions should not interfere with the successful predictions of the SM, and new particles predicted by the models must be heavier than the corresponding experimental lower bounds on masses. The new particles include new vector bosons corresponding to the new generators of the extended gauge symmetry, in general also new fermions, such as, heavier replica of known fermions, heavy neutrinos and, e.g., so-called mirror fermions [66], as well as new Higgs scalars.

3.1   Extended Gauge Models for Massive Neutrinos

Possible models for massive neutrinos can be divided into two categories, according to whether the neutrinos are Majorana or Dirac particles. In the category of Majorana neutrinos there are models where the light neutrino masses are generated via the see-saw mechanism [33], as well as models where the neutrino masses are induced by radiative corrections [67].

The see-saw mechanism is the simplest way to understand the lightness of the left-handed neutrinos, and it is realized in many extended models. A central ingredient of this mechanism is a Majorana mass term of the right-handed neutrinos, which is much larger than any mass scale of the SM. As for all mass parameters, the origin of this mass should be connected to some symmetry breaking taking place in the theory. The models with this property include the local left-right symmetric model, local $SO(10)$ based models [34], models with local or global horizontal symmetry [68, 69] and models with global Peccei-Quinn symmetry [70].

In another class of models the left-handed neutrino achieves a Majorana mass

through radiative corrections involving a scalar particle that mediates lepton number violating interactions [71]. This scalar is not allowed to obtain vacuum expectation value, since it would break the existing electroweak symmetry incorrectly. The induced neutrino mass is typically $m_{\nu_\ell} = f^2 m_\ell$, where m_ℓ is the mass of the charged lepton and f is the Yukawa coupling of the charged lepton.

The models where neutrinos are Dirac particles are in general unnatural in the sense that the smallness of the neutrino mass should be put in by hand, or one has to allow a large hierargy ($\geq 10^6$) between the Yukawa couplings of charged leptons and neutrinos. In the latter case the right-handed neutrinos can be included as singlets as can be done in the $SU(2)_L \otimes U(1)_Y$ symmetric models. However, it is possible to construct models where we have see-saw like mechanism for massive Dirac neutrinos. This can be done e.g., in a class of models containing an $SU(n)_F$ family symmetry broken by Higgs scalars transforming according to the fundamental representation of the gauge symmetry [72].

The models where the heavy neutrinos are mirror particles [66] are also considered in this Thesis. Mirror fermions are particles which couple to the ordinary W boson in $V + A$ currents, in contrast with ordinary fermions which couple in $V - A$ currents. Mirror fermions arise naturally in so-called family unified models based on large orthogonal groups [73], as well as in a class of models based on large unitary groups [74] or on exceptional groups [75]. Also in the Kaluza-Klein theories [76] and in some composite models [77] the existence of the mirror fermions is predicted.

The lower bound of charged mirror fermion masses is set by the undiscovery of these particles at LEP, and it is $m_F \gtrsim \mathcal{O}(45)$ GeV. The lower bound on mirror neutrino masses depend heavily on the mixing of these particles and therefore there is no generally valid limit available. On the other hand, they must be lighter than $\mathcal{O}(300)$ GeV. This is due to the fact that their masses are generated in the spontaneous symmetry breaking of $SU(2)_L \otimes U(1)_Y$ gauge symmetry and are therefore limited, if one wants to maintain the validity of perturbation theory by not allowing too large Yukawa couplings. Even if mirror fermions were too heavy to be observed, they may have manifest effects through possible mixings with the ordinary particles. As a result of such a mixing the gauge interactions of the ordinary particles would be a mixture of

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the $V - A$ and $V + A$ interactions. Our analysis in Papers I and II takes this possibility into account by allowing for a general V, A structure of interactions.

3.2   Left-Right Symmetric Model and Massive Neutrinos

The original motivation of the left-right symmetric model based on an $SU(2)_R \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry is related to the maximal parity violation observed in the low-energy weak interactions. In the SM, parity violation is arranged quite artificially by treating left- and right-handed fermions on unequal footing. In the left-right symmetric model instead, the parity violation has a dynamical origin. The left-handed and right-handed fields are treated on the same basis in Lagrangian, but the vacuum state is not invariant under the parity transformation. The parity violation is related to the spontaneous breaking of the left-right symmetry below some energy scale considerably larger than the electroweak scale.

In the left-right symmetric model the $U(1)$ generator has a clear physical meaning: it is the baryon number operator B minus the lepton number operator L [78]. As this symmetry is spontaneously broken, there exist lepton and baryon number violating interactions. The lepton number violation manifests itself most clearly in neutrino physics, e.g., in the so-called neutrinoless double-β decay [79]. Lepton number violating processes could also play a crucial role in explaining the baryon number asymmetry of the universe [80].

The particle spectrum of the left-right symmetric model contains all the SM particles, added with the right-handed neutrinos, and new heavier gauge bosons W_2 and Z_2 associated with the $SU(2)_R$ symmetry as well as several new Higgs bosons. The decomposition of the scalar sector depends on the symmetry breaking chain. One usually considers a model where the Higgs sector consists of one bidoublet Higgs and so-called left- and right-handed triplet Higgses [81]. In some versions of the model the triplets are replaced by two doublet fields, or there are both doublets and triplets. In the most simple model, the Higgs sector consists of one bidoublet field and one right-handed triplet field, enough to break the symmetry appropriately and to generate the fermion masses. In this Thesis we will assume that both the left- and right-handed triplet fields

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are present.

Under the $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$ gauge symmetry each fermion generation is assigned according to $(k = 1, 2, \ldots)$

$$
L_{kL} = \begin{pmatrix} \nu_{\ell_k} \\ \ell_k \end{pmatrix}_L \sim (1, 2, -1), \qquad L_{kR} = \begin{pmatrix} \ell_k \\ \nu_{\ell_k} \end{pmatrix}_R \sim (2, 1, -1),
$$

$$
Q_{kL} = \begin{pmatrix} u_k \\ d_k \end{pmatrix}_L \sim (1, 2, \tfrac{1}{3}), \qquad Q_{kR} = \begin{pmatrix} u_k \\ d_k \end{pmatrix}_R \sim (2, 1, \tfrac{1}{3}). \tag{35}
$$

This is a natural extension to the SM particle spectrum (1) treating left- and right-handed fermions equally.

The breaking of the $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$ gauge symmetry to the electromagnetic gauge symmetry $U(1)_{em}$ proceeds through two stages. The break down chain is

$$
SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L} \xrightarrow{M_R} SU(2)_L \otimes U(1)_Y \xrightarrow{M_{EW}} U(1)_{em}. \tag{36}
$$

The first spontaneous breaking at the scale M_R is due to the right-handed triplet Higgs field $\Delta_R \sim (3, 1, +2)$, and the spontaneous breaking at the electroweak scale M_{EW} is due to the bidoublet $\phi \sim (2^*, 2, 0)$ and the left-handed triplet field $\Delta_L \sim (1, 3, +2)$. The left- and right-handed triplet and the bidoublet fields are:

$$
\Delta_{L/R} = \begin{pmatrix} \frac{1}{\sqrt{2}} \Delta^+_{L/R} & \Delta^{++}_{L/R} \\ \Delta^0_{L/R} & -\frac{1}{\sqrt{2}} \Delta^+_{L/R} \end{pmatrix}, \qquad \phi = \begin{pmatrix} \phi^0_1 & \phi^+_1 \\ \phi^-_2 & \phi^0_2 \end{pmatrix}, \tag{37}
$$

which at the spontaneous symmetry breaking acquire the following vacuum expectation values:

$$
\langle \Delta_{L/R} \rangle = \begin{pmatrix} 0 & 0 \\ v_{L/R} & 0 \end{pmatrix}, \qquad \langle \phi \rangle = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 e^{i\alpha} \end{pmatrix}, \tag{38}
$$

where $v_{L/R}$, κ_1, κ_2 and α are real parameters.

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At the first stage of the symmetry breaking the right-handed vector bosons W_R and Z_R^0 acquire the masses

$$M_{W_R} = g v_R, \qquad M_{Z_R} = M_{W_R} \sqrt{\frac{2 \cos^2 \theta_W}{\cos 2\theta_W}}, \qquad (39)$$

where θ_W is the weak mixing angle of the left-right symmetric model. The relation (39) is valid only if we assume $g_L = g_R$, i.e. that the gauge coupling constants of the left- and right-handed sectors are equal. This is a common assumption, but in most of the results presented in this Thesis, it is not made.

The vacuum expectation values of the left-handed triplet and the bidoublet fields force the symmetry breaking at the electroweak scale. The experimental results [50] on the parameter

$$\rho_0 = \left(\frac{M_W}{\cos \Theta_W M_Z}\right)^2 = \frac{\sum_{\text{isospin}} \left\{[T^i(T^i + 1) - (T_3^i)^2] \langle H_i\rangle\right\}}{\sum_{\text{isospin}} \left\{2(T_3^i)^2 \langle H_i\rangle\right\}}, \qquad (40)$$

where T and T_3 are the $SU(2)_L$ isospin and its third component, respectively, and the sum goes over the Higgs fields H_i that acquire a non-vanishing vacuum expectation value, imply that the vacuum expectation values κ_1, κ_2 and v_L must obey $v_L \ll \kappa_1, \kappa_2$. Therefore, the main contribution to the mass of the left-handed boson comes from the vacuum expectation value of the bidoublet field.

Since the bidoublet field ϕ transforms nontrivially under both the $SU(2)_L$ and $SU(2)_R$ groups, the left- and right-handed gauge bosons may mix with each other. The physical eigenstates of the vector bosons are hence in general given by

$$\begin{aligned}
W_1 &= \cos\zeta W_L + e^{i\alpha} \sin\zeta W_R \simeq W_L, \\[4pt]
W_2 &= -e^{i\alpha} \sin\zeta W_L + \cos\zeta W_R \simeq W_R, \\[4pt]
A &= \sin\theta_W (W_L^3 + W_R^3) + \sqrt{\cos 2\theta_W}\, B \equiv \gamma, \\[4pt]
Z_L &= \cos\theta_W W_L^3 - \sin\theta_W \tan\theta_W W_R^3 - \tan\theta_W \sqrt{\cos 2\theta_W}\, B \simeq Z_1, \\[4pt]
Z_R &= \sec\theta_W \sqrt{\cos 2\theta_W}\, W_R^3 - \tan\theta_W B \simeq Z_2
\end{aligned} \qquad (41)$$

where A is the massless gauge field of the unbroken $U(1)_{em}$ symmetry, the photon, and

$$\tan\zeta = \frac{\kappa_1 \kappa_2}{\kappa_1^2 + \kappa_2^2 + 8 v_R^2}. \qquad (42)$$

The masses of the light weak bosons are given approximately by

$$M_{W_L}^2 \;\simeq\; \tfrac{1}{2}g^2\,(\kappa_1^2 + \kappa_2^2) \equiv M_{W_1}^2\,,$$

$$M_{Z_L}^2 \;\simeq\; M_Z^2\,[1 - \eta A_W] + \mathcal{O}(\eta^2)\,,$$

(43)

where $A_W \equiv \tfrac{1}{2}\cos 2\theta_W(1 - \tfrac{1}{4}\tan^4\theta_W)$ and $\eta = M_{W_L}^2/M_{W_R}^2$. By comparing equations (43) and (10) one can see that M_{Z_L} is always less than the mass M_Z of the Z-boson of the SM.

The charged and neutral current Lagrangians are given by

$$\mathcal{L}_{wk}^{CC} \;\simeq\; \frac{g}{2\sqrt{2}}\Big[\Big(\cos\zeta\, J_{L\,\mu}^+ + e^{i\alpha}\sin\zeta\, J_{R\,\mu}^+\Big) W_L^{+\mu}$$

$$+ \Big(\cos\zeta\, J_{R\,\mu}^+ - e^{i\alpha}\sin\zeta\, J_{L\,\mu}^+\Big) W_R^{+\mu} + \text{h.c.}\Big]$$

$$\mathcal{L}_{wk}^{NC} \;=\; e J_\mu^{em} A^\mu + \frac{g}{\cos\theta_W}\Big\{\Big[J_{L\,\mu}^Z - \eta\cos\theta_W\Big(\sin^2\theta_W J_{L\,\mu}^Z + \cos^2\theta_W J_{R\,\mu}^Z\Big)\Big]Z_L^\mu$$

$$+ (\cos^2\theta_W)^{-1/2}\Big(\sin^2\theta_W J_{L\,\mu}^Z + \cos^2\theta_W J_{R\,\mu}^Z\Big)Z_R^\mu\Big\}\,,$$

(44)

where the weak neutral currents are $J_{L/R}^Z = J_{L/R}^3 - Q\sin^2\theta_W\, J^{em}$ and the J^{em} is the electromagnetic current. The observed $V-A$ form of the weak interactions at low energies follows from the condition $\eta \ll 1$ and $\zeta \ll 1$ [82].

The most general Yukawa coupling Lagrangian of leptons is

$$\mathcal{L}_Y = \bar{L}_{iL}(h_{ij}\phi + \tilde{h}_{ij}\tilde{\phi})L_{jR} + (f_L)_{ij}\bar{L}_{iR}^c(i\tau_2)\vec{\tau}\, L_{jL}\cdot\vec{\Delta}_L + (f_R)_{ij}\bar{L}_{iL}^c(i\tau_2)\vec{\tau}\, L_{jR}\cdot\vec{\Delta}_R + \text{h.c.} \quad (45)$$

where h, $\tilde{h}$, and $f_{L(R)}$ are the $n \times n$ Yukawa coupling matrices (n is the number of lepton generations) and $\tilde{\phi} = \tau_2\phi^*\tau_2$ is the conjugated bidoublet field. The most general Yukawa coupling Lagrangian of quarks is similar to (45), but there are no couplings to the triplet fields because of the $B-L$ symmetry.

3.2.1   See-saw Mechanism

Let us consider more closely the generation of neutrino masses in the $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$ model. According to eq. (45), neutrino fields acquire in the symmetry breaking Dirac masses due to their coupling with the bidoublet Higgs ϕ,

$$\mathcal{L}_{m_\nu}^D = \Big(h_{ij}\langle\phi_1^0\rangle + \tilde{h}_{ij}\langle\phi_2^{0*}\rangle\Big)\bar{\nu}_{iL}\nu_{jR} + \text{h.c.}\,, \quad (46)$$

and Majorana masses due to their coupling with the triplet Higgses Δ_L and Δ_R,

$$\mathcal{L}_{m_\nu}^M = (f_L)_{ij} \langle \Delta_L^0 \rangle \bar{\nu}_{iR}^c \nu_{jL} + (f_R)_{ij} \langle \Delta_R^0 \rangle \bar{\nu}_{iL}^c \nu_{jR} + \text{h.c.}. \tag{47}$$

The complete mass Lagrangian can thus be written in the form

$$\mathcal{L}_{m_\nu} = (\bar{\nu}_{iR}^c \; \bar{\nu}_{iR}) \begin{pmatrix} (f_L^*)_{ij} \langle \Delta_L^{0\,*} \rangle & \frac{1}{2} \left(h_{ij}^* \langle \phi_1^{0\,*} \rangle + \tilde{h}_{ij} \langle \phi_2^0 \rangle \right) \\ \frac{1}{2} \left(h_{ji}^* \langle \phi_1^{0\,*} \rangle + \tilde{h}_{ji} \langle \phi_2^0 \rangle \right) & (f_R)_{ij} \langle \Delta_R^0 \rangle \end{pmatrix} \begin{pmatrix} \nu_{jL} \\ \nu_{jL}^c \end{pmatrix} + \text{h.c.}, \tag{48}$$

where we have used the relation $\bar{\nu}_R \nu_L = \frac{1}{2} \left(\bar{\nu}_R \nu_L + \bar{\nu}_R^c \nu_L^c \right)$. In order to find the physical neutrino fields one has to diagonalize this Lagrangian.

As already mentioned, the experimental constraints require that $\langle \Delta_R^0 \rangle \gg \langle \phi_{1,2}^0 \rangle \gg \langle \Delta_L^0 \rangle$. It is therefore reasonable to approximate $(f_L^*)_{ij} \langle \Delta_L^{0\,*} \rangle \simeq 0$ in (48). The mass Lagrangian then obtains the following see-saw form:

$$\mathcal{L}_{m_\nu} \approx (\bar{\nu}_R^c \; \bar{\nu}_R) \begin{pmatrix} 0 & m_D \\ m_D^T & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} + \text{h.c.} \equiv (\bar{\nu}_R^c \; \bar{\nu}_R) \, M \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} + \text{h.c.}, \tag{49}$$

where the expressions of the 3×3 matrices m_D and m_R can be read from (48). The left-handed neutrino fields appearing here can be identified with the left-handed neutrinos ν_{eL}, $\nu_{\mu L}$ and $\nu_{\tau L}$. In the case of one neutrino flavor the mass Lagrangian can be diagonalized with an unitary matrix transformation U in the following way:

$$U^\dagger M U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \widehat{m}_1 & 0 \\ 0 & \widehat{m}_2 \end{pmatrix} = M'. \tag{50}$$

The eigenvalues of the matrix M' $(m_R \gg m_D)$ are:

$$\widehat{m}_{1,2} = \frac{1}{2} m_R \left[1 \pm \sqrt{1 + \left(\frac{m_D}{m_R} \right)^2} \right] \simeq \begin{cases} m_R \\ -\dfrac{m_D^2}{m_R} \end{cases}. \tag{51}$$

They are related to the masses of the physical neutrinos via $m_i = \widehat{m}_i \eta_i^{CP}$, where $\eta_i^{CP} = \pm 1$ is the CP parity of the neutrino fields. The corresponding mass eigenstates

χ_1 and χ_2 are given by

$$
\begin{aligned}
\chi_1 &\simeq \nu_L + \nu_R^c - \frac{m_D}{m_R}\left(\nu_R + \nu_L^c\right), \\
\chi_2 &\simeq \frac{m_D}{m_R}\left(\nu_R + \nu_L^c\right) - \nu_R + \nu_L^c.
\end{aligned}
\tag{52}
$$

As seen from this, the physical neutrinos χ_i are Majorana particles, i.e. they fulfill the Majorana condition $\chi_i = \chi_i^c$, $i = 1, 2$. The neutrino mixing angle θ is given by $\tan 2\theta \simeq 2m_D/m_R$. The current sates of the neutrino fields in terms of the mass eigenstates are

$$
\begin{aligned}
\nu_L &\simeq -\chi_{2L} + \frac{m_D}{m_R}\chi_{1L}, \\
\nu_L^c &\simeq \chi_{1L} + \frac{m_D}{m_R}\chi_{2L}.
\end{aligned}
\tag{53}
$$

That is, the left-handed neutrino ν_L appearing in low-energy phenomena corresponds to the light mass eigenstates χ_2 and the right-handed neutrino ν_R (ν_L^c) corresponds to the heavy mass eigenstate χ_1.

3.3   Heavy Neutrino Physics at Future e^+e^- Colliders

Future high-energy e^+e^- colliders will offer a clean environment for investigating the properties of heavy neutrinos. This is an important issue since heavy neutrinos are one of the main characteristics of many extended gauge models that give the SM as a low-energy approximation.

In Papers I and II we investigate the production of heavy neutrinos in e^+e^- collisions. In Paper I the main focus is on the collision energies $\sqrt{s} = 150 - 300$ GeV, i.e. in the energy range of the LEP200 and a low energy linear collider. The production cross sections of the pair production of heavy neutrinos ($e^+e^- \rightarrow \overline{N}_1 N_2$) and a single heavy neutrino production ($e^+e^- \rightarrow \overline{N}_1 \nu_2$ or $N_1 \overline{\nu}_2$), as well as the decay properties of heavy neutrinos, are analyzed. As a model for weak interactions we use phenomenological model which has one set of weak vector bosons with general (real) $V \pm A$ couplings. The choice of just one vector boson generation is justified by the experimental results [82, 83], that the mass of the heavier vector bosons is much higher than the chosen collision energy scale. In our analysis both Majorana and Dirac cases

are studied and especially the differences between heavy Dirac and Majorana neutrino production, as well as the differences between the $V + A$ and $V - A$ currents, are investigated.

The results we obtained in Paper I show that the pair production rate of heavy Majorana neutrinos is smaller than the corresponding rate of Dirac neurinos with the same mass. The total pair production rate is the same for both left- and right-handed neutrinos, but the angular distributions are different and one should be able to use forward-backward asymmetries for distinguishing right- and left-handed neutrinos provided one has an opportunity to collect enough events. The angular distributions can be used also to distinguish the Dirac and Majorana neutrinos.

In the single heavy neutrino production the Majorana neutrino production rate is two times bigger that the Dirac neutrino production rate. Provided the coupling constants are equal for both $V + A$ and $V - A$ currents, the production rates are typically by a factor of 1.5 larger in the case of $V + A$ currents. Since the angular distributions are dramatically different in the single heavy neutrino production the separation of Dirac and Majorana neutrinos, as well as the right- and left-handed neutrinos, is possible with considerably lower statistics than in the pair production. The experimental signal for single heavy neutrino production consists of heavily unbalanced events with two charged leptons or unbalanced hadron jets accompanied with one charged lepton. These very spectacular events should make the detection of heavy neutrinos easy because the expected background is small.

In Paper I the subsequent decays of the heavy neutrinos are also studied. The results verify the well known fact that the total decay width of a Majorana neutrino is two times bigger that of the Dirac neutrino with the same mass and the same couplings. The angular distributions of leptons produced in the decay are also different for Dirac and Majorana neutrinos. Based on these two properties we propose a powerful and experimentally sound method to distinguish the Majorana and Dirac cases via studying the correlated production of the same sign lepton pairs from the $\overline{N}N$ system. Also the differences in angular distributions and lifetimes can be used for distinguishing heavy Dirac and Majorana neutrinos, but it would require higher statistics.

In Paper II the production of heavy Dirac and Majorana neutrinos in e^+e^- collisions $(e^+e^- \rightarrow \overline{N}_1 N_2)$ are studied using a gauge model with an arbitrary number of neutral vector bosons Z_i^0, $i = 1, \ldots, N$ and charged vector bosons $W_j^{\pm}$ $j = 1, \ldots, M$ with the most general V, A couplings. The production of heavy neutrinos in Higgs exchange channels are also analyzed since in specific models some of the Yukawa couplings could be large enough in these channels to give measurable contribution to the production rate. Numerical results are presented for the case of a minimal left-right symmetric model with two vector boson generations and with a minimal set of Higgs fields, i.e. with two triplets and one bidoublet. The model is simplified by using a phenomenologically motivated set of gauge and Yukawa couplings. A similar analysis with a more strictly defined version of the left-right symmetric model is presented in [84].

The main issues of Paper II are to study carefully the interference patterns between different production channels, as well as to find differences between left- and right-currents and between Dirac and Majorana neutrinos. By separating the interference terms we found that they can cause big changes to the heavy neutrino production threshold behaviour, as well as to the production rates in the vicinity of the heavy boson pole. The interference terms are non-negligible although some of the interfering channels are much smaller that the other channels. In the model specified in Paper II some 50 to 100 heavy Dirac or Majorana neutrinos with a mass $m_N = 150$ GeV are expected to be produced annually at a 500 GeV linear collider assuming a luminosity $\mathcal{L}_{\text{year}} = 10$ fb^{-1}. It is also shown that, in order to make a reliable distinction between Dirac and Majorana cases, higher statistics would be required. The possibility to do production threshold scans would also make the distinction of the Dirac and Majorana neutrinos easier, but for this one would need a linear collider with adjustable electron and positron beam energies and a very flexible focusing systems.

Concerning the angular distributions and even the decays of the heavy neutrinos, the arguments presented for Paper I are also valid for Paper II. There are also differences due to the more general model used in Paper II, e.g, in the pair production of heavy neutrinos, the pole of the second neutral boson Z_2 enhances the pair production rate drastically. This is due to the fact that heavy neutrinos tend to have

non-suppressed coupling to the heavier bosons. This is not true in the single heavy neutrino production since either the heavy neutrino or the light neutrino couplings to the charged vector bosons are suppressed. One should also note that among all heavy neutrino pair production channels the Higgs boson exchange processes give always the smallest contribution. On the other hand, the results in Paper II also indicate that the production of heavy neutrinos produced via a single charged triplet Higgs exchange would be visible only at high collision energies ($\sqrt{s} = 1 - 2$ TeV) or at e^+e^- linear collider with a very high luminosity ($\mathcal{L}_{\text{year}} \geq 100$ fb^{-1}).

4   Conclusions

Although high-energy physics experiments have increased our knowledge about particle physics interactions and phenomena to a remarkable level, it is by no means excluded that new phenomena are found when one goes beyond the energy and precision reach of our present facilities. Especially the future collider experiments, which aim to the TeV energy scale, have a potential to reveal phenomena unknown at present. It will be of great importance to carefully study in advance the new physics topics one could explore with these next generation facilities. The aim of this Thesis has been to study the neutrino physics phenomenology at the future e^+e^- colliders (research Papers I, II and III) and to investigate the phenomenology of the most poorly known Standard Model constituents, the top quark and the tau neutrino, at e^+e^- colliders (Papers III and IV).

The goal of Paper III is to find a method to observe the only directly undiscovered fermion of the SM, the tau neutrino. The proposed method consists of two parts. The first part is a very asymmetric e^+e^- collider which runs either at the Z^0-pole, i.e. at $\sqrt{s} = M_{Z^0}$, or at $\sqrt{s} \approx 4.2$ GeV, i.e. just above the τ-lepton pair production threshold. The second part consist of a simple rejection detector (veto-trigger) around the interaction point and of a coarsely instrumented muon spectrometer with a large volume and mass. The asymmetric collider could be built by using a positron beam of the TeV range linear collider as a high energy beam, and the characteristics of the low energy electron beam could be similar to those of electron damping rings needed for the future e^+e^- linear colliders. Using the realistic accelerator and detector setup proposed in Paper III, the discovery signal is found to be quite marginal. While making the discovery of the tau neutrino possible the apparatus would not provide a possibility to a very systematic study of the properties of the produced tau neutrinos. On the other hand, with these minimal specifications one should be able to build the whole apparatus, alongside an existing linear collider, with a fraction of the cost of a stand

alone experiment with specialized accelerator and detectors.

The single top production considered in Paper IV will be an important process in the next generation e^+e^- linear colliders. According to present knowledge, at LEP200 the cross section will be, however, too small to yield an observable signal, in contrast to the more optimistic estimates presented in Paper IV.

One interesting prediction common to many extensions of the SM is the existence of massive neutrinos. The main attention in this Thesis is paid on the left-right symmetric model which produces via the see-saw mechanism in each generation a light and a heavy neutrino which are Majorana particles. The observation of a heavy neutrino in e^+e^- collisions would prove that the SM is not the final theory, since it does not have such particles. Due to their small mass, it is very difficult to probe the nature of light neutrinos in scattering experiments. In the processes involving heavy neutrinos, instead, the signals which would make distinction between Dirac and Majorana neutrinos would be easier to detect. The results presented in Paper I and, especially, in Paper II should facilitate such investigations by giving the relevant cross section and decay widths in a form which applies to a large class of extended models.

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